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Effect of initial stress on harmonic plane homogeneous waves in viscoelastic anisotropic media

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Abstract

Biot's theory is used to study the effect of initial stress on phase velocity and attenuation of plane homogeneous waves in viscoelastic anisotropic medium. Modified Christoffel equations are derived for three-dimensional wave propagation in a general viscoelastic anisotropic medium under initial stress. Effect of initial stress on wave propagation is observed through the deviations in phase velocity and attenuation–amplitude factor for each of the three existing quasi-waves. A particular numerical example shows that the attenuation is more sensitive to the presence of initial stress as compared to the propagation velocities.

1. Introduction

Wave propagation in viscoelastic anisotropic medium in the presence of initial stress is of great interest in geophysics and other branches of applied sciences such as petroleum engineering, biological sciences, electronics, electric engineering and seismology. In civil engineering and geophysics, the problems of consolidation and tectonics involve earth masses that are initially under high stress. The folds and fractures in the sedimentary layers are the result of differential stress environment in the sediments. In the problems of foundation engineering, the influence of initial stress appears in a buoyancy effect, which amounts to floating a building on its foundation. The initial stress state inside the earth is mainly due to the pressure of overburden. Crustal rocks are always subjected to stresses. The slow process of creep inside the earth produces the initial stress, which may be approximated as homogeneous near the surface (Biot [1]). Stress differences between lithosphere and upper mantle has been indicated in the experimental studies of McGarr [2] and Hanks and Raleigh [3]. In perhaps, the earliest effort Cauchy [4] assumed that the initial stress was due to the central forces between the particles of solid. A definitive theory explaining the elastodynamics of a body under initial stress was developed by Biot [1,5]. An elegant and elaborate exposition of this theory is found in Biot [6]. Hayes [7] studied the velocities of wave propagation in pre-stressed elastic solids. A large number of studies in the later years improved the understanding of wave propagation characteristics of materials under initial stress. Tolstoy [8] studied the effect of gravity and hydrostatic pressure on velocities of elastic waves. Man and

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Lu [9] developed a new theory, which is applicable to both applied and residual stresses. Degtyar and Rokhlin [10] discussed analytically the effect of initial stress on wave propagation through an anisotropic/anisotropic interface. They used the stress-dependent orthotropic elastic coefficients from the study by Man and Lu [9]. Sharma and Neetu [11] analyzed analytically as well as numerically the wave velocities in a pre-stressed anisotropic elastic medium. Actually, stresses affect not only the velocities of waves but also their attenuation. One obvious failure of the perfectly elastic medium is that there is no attenuation. The theory of visco-elasticity is important for consideration of the attenuation of stress waves and the damping of vibrations. Due to creep deformation, crustal rocks can be modeled as viscoelastic solids pervaded by aligned cracks. The preferential alignments in the Earth ranging from mineral orientations, grains, or micro-cracks to regional fractures result in the seismic anisotropy. These alignments are the result of differential environment in the crust. Many hydrocarbon reservoirs lie beneath dipping clastic sequences, and these clastics possess anisotropy. This implies the coexistence of viscoelastic anisotropy as well as initial stress in the crust. Biot [6] presented the theory of elasticity and viscoelasticity of initially stressed solids and fluids including applications to finite strain. The study of waves in an initially stressed medium is of interest not for theoretical taste only but for practical purposes too. A dynamical explanation of earthquake phenomena is required in earthquake research. In this context, the term dynamics implies a consideration of the initial stress with in the viscoelastic Earth that act to cause fault ruptures and ground displacements. Studies of the past earthquakes equipped with other geophysical data are helpful to find out the location and size of the future earthquake. However, most of the research does not take into account the effects of initial stress on the wave propagation in viscoelastic medium. Cooper [12] studied the reflection and transmission of oblique plane waves at an interface between viscoelastic media. Biswas [13] studied the transmission of SH waves through a viscoelastic layer embedded between two elastic half-spaces and concluded that the effect of imaginary components of elastic properties on the transmission of waves increases as the frequency increases. Shaw and Bugl [14] studied the transmission of time harmonic plane P and SV waves in a layered infinite linear viscoelastic medium. Cederbaum and Aboiudi [15] studied the dynamic response of viscoelastic-laminated plates. Pal and Kumar [16] analyzed the generation and propagation of SH-type waves due to stress discontinuity in a linear viscoelastic layered medium. Nygren et al. [17] discussed the dissipation of wave energy in a viscoelastic junction between elastic bars and its dependence on transmission direction. Optimization of viscoelastic junctions with regard to transmission of wave energy was also discussed by Nygren et al. [18]. However, the most significant and generalized work was that of Červený and Pšenčik [19]. In this paper, they discussed, in detail, the plane waves in viscoelastic anisotropic media. They studied the properties of homogeneous and inhomogeneous plane waves propagating in an unbounded viscoelastic anisotropic medium in an arbitrarily specified direction. They established the fact that to determine the complex-valued slowness vectors of such plane waves and their polarization vectors, individual approaches differ in the way in which the slowness vector of the plane wave under consideration is specified. They introduced the following three different specifications for the slowness vector: the directional specification, the componental specification and the mixed specification.

The presence of initial stress in each type of realistic material whether in any form motivates to the introduction of initial stress in the viscoelastic anisotropic media. Wave velocities and attenuation are the two important propagation properties, which provide information about the saturation and structure of in situ rocks. The presence of initial stress affects the velocities and attenuation of waves in a medium. A large quantity of initial stress may develop in a medium due to many physical causes. Keeping it in mind, it should be of considerable interest to study the propagation of plane homogeneous waves considering the medium to be initially stressed. The author proposes to study the effects of initial stress on velocity and attenuation of plane homogeneous waves in viscoelastic medium which undoubtedly will be helpful for seismologists, geologists, mechanical and electrical engineers in estimating significant results in their respective fields and in implication of their conclusion for the development of their technologies. This study enables geophysicists to determine the seismic structure of the oceanic crust and to determine gross earth structures, specifically, the crustal and upper mantle structures. The problem studied here will be helpful in modeling the construction of an earthquake resistant design and dams to control the transmission of earthquake energy to the upper surface in order to avoid the destruction. In fact, the earth behaves as an initially stressed viscoelastic model. Thus, this attempt is to study the wave propagation in a more realistic model of the earth's surface. This study may be useful for the detection of viscoelastic materials in earth's crust.

2. Viscoelastic anisotropic medium under initial stress

2.1. Equation of motion

Consider a general viscoelastic anisotropic medium under homogeneous initial stress S_{ij}. Following Biot [6], the equations for wave motion in this medium, in the absence of body force, are given by

$$s_{ij,j} + S_{jk}\omega_{ik,j} + S_{ik}\omega_{jk,j} = \rho \ddot{u}_i, \tag{1}$$

where u_i are displacement components. The components of rotation are given by $\omega_{ij} = (u_{i,j} - u_{j,i})/2$. Indices can take values of 1,2,3. The comma before an index represents (partial) space differentiation and dot denotes (partial) time derivative. Repeated index implies summation. The incremental stresses s_{ij} in the medium are expressed as

$$s_{ij} = B_{ijkl} u_{k,l}, \tag{2}$$

where the fourth rank tensor $B_{ijkl} (= B_{jikl} = B_{ijlk})$ represents the complex-valued, frequency dependent, viscoelastic moduli. ρ is the density of the medium. Another property, given by

$$B_{ijkl} - B_{klij} = S_{kl}\delta_{ij} - S_{ij}\delta_{kl} \tag{3}$$

ensures the existence of energy density function for the medium (Biot [5,6]), where δ_{ij} is Kronecker delta.

2.2. Plane wave propagation

To seek the harmonic solution of (1), for the propagation of plane homogeneous waves, write

$$u_j = U_j \exp\{-\imath\omega(t - p_n x_n)\} \quad (j = 1, 2, 3), \tag{4}$$

where u_j, p_j, U_j are Cartesian components of the complex-valued displacement vector $\mathbf{u}$, slowness vector $\mathbf{p}$ and polarization vector $\mathbf{U}$, respectively. Moreover, t is time and ω is a fixed, positive circular frequency.

Substituting (4) in (1) yields a system of three homogeneous equations, given by

$$(X_{ik} - h\delta_{ik})U_k = 0, \quad (i = 1, 2, 3), \tag{5}$$

where, for row matrix $\mathbf{p} = (p_1, p_2, p_3)$ and its transpose $\mathbf{p}^{\mathrm{T}}$,

$$h = 1 - \mathbf{pSp}^{\mathrm{T}}/\rho \tag{6}$$

and X_{ik} are the elements of a matrix

$$\mathbf{X} = [\mathbf{Z} - \mathbf{S} + \mathbf{Sp}^{\mathrm{T}}\mathbf{p} - \mathbf{p}^{\mathrm{T}}\mathbf{pS}]/\rho. \tag{7}$$

The matrix $\mathbf{S} = \frac{1}{2}\{S_{ij}\}$ and the matrix $\mathbf{Z}$ is as defined in Appendix A. However, the matrix $\mathbf{Z}$ is not a symmetric matrix but due to relations (3), the matrix $\mathbf{X}$ is a symmetric one. System (5) may be called the modified Christoffel equations.

3. Phase velocity

Non-trivial solution of modified Christoffel Eq. (5) requires satisfying an equation, given by

$$\det(\mathbf{X} - h\mathbf{I}) = 0, \tag{8}$$

where $\mathbf{I}$ is the identity matrix of order three. In viscoelastic media, slowness vector $\mathbf{p}$ is complex valued, $\mathbf{p} = \mathbf{P} + \imath\mathbf{A}$. Here $\mathbf{P}$ is the real-valued propagation vector, perpendicular to the plane of constant phases, and $\mathbf{A}$ is the real-valued attenuation vector, perpendicular to the plane of constant amplitudes, oriented in the direction of the maximum decay of amplitude. For $\mathbf{P}$ parallel to $\mathbf{A}$, the plane wave is called homogeneous, and for $\mathbf{P}$ and $\mathbf{A}$ non-parallel, it is called inhomogeneous. The procedures of determining the slowness vector $\mathbf{p}$ of a plane homogeneous wave propagating in a viscoelastic anisotropic medium under the effect of initial stresses, satisfying (5) and (8), are discussed in this paper. We also introduce the real-valued unit vectors $\mathbf{N}$ and $\mathbf{M}$ in the directions of $\mathbf{P}$ and $\mathbf{A}$ (for plane homogeneous waves, $\mathbf{P}\|\mathbf{A} \Rightarrow \mathbf{N}\|\mathbf{M}$), phase velocity v, attenuation-propagation ratio δ and attenuation (or inhomogeneity) angle γ, as

$$\mathbf{N} = \frac{\mathbf{P}}{|\mathbf{P}|},\ \mathbf{M} = \frac{\mathbf{A}}{|\mathbf{A}|}, v = \frac{1}{|\mathbf{P}|}, \delta = \frac{|\mathbf{A}|}{|\mathbf{P}|},\ \cos\gamma = \mathbf{N}\bullet\mathbf{M}. \tag{9}$$

The plane waves are called homogeneous for $\gamma = 0$.

4. Directional specification of the slowness vector

We shall use the directional specification of the slowness vector $\mathbf{p}$, in which $\mathbf{p}$ is expressed in terms of known real-valued unit vectors $\mathbf{N}$ and $\mathbf{M}$,

$$\mathbf{p} = \frac{\mathbf{N} + \imath\delta\mathbf{M}}{v}. \tag{10}$$

For homogeneous plane waves, $\gamma = 0$. So, $\mathbf{N}\equiv\mathbf{M}$ and (10) yields

$$\mathbf{p} = \frac{1 + i\delta}{v}\mathbf{N}. \tag{11}$$

Eq. (6) can be written as

$$v = (1 + i\delta)\sqrt{h + \frac{\mathbf{NSN}^{\mathrm{T}}}{\rho}}, \tag{12}$$

where

$$\mathbf{N} = (n_1, n_2, n_3) = \frac{v}{1 + \imath\delta}(p_1, p_2, p_3).$$

Eq. (5) can be written as

$$\left[X_{ik} - \left(\frac{v^2}{(1 + i\delta)^2} - \frac{\mathbf{NSN}^{\mathrm{T}}}{\rho}\right)\delta_{ik}\right]U_k = 0 \quad (i = 1, 2, 3), \tag{13}$$

where

$$X_{ik}(N_n) = \frac{\mathbf{Z} - \mathbf{S} + \mathbf{SN}^{\mathrm{T}}\mathbf{N} - \mathbf{N}^{\mathrm{T}}\mathbf{NS}}{\rho} \tag{14}$$

is the complex-valued Christoffel matrix. Let us emphasize that B_{ijkl} are complex-valued, but N_i real valued. For a known model (B_{ijkl}) and known direction of propagation $\mathbf{N}$, it is simple to calculate $X_{ik}(N_n)$.

The condition of the solvability of (13) is

$$\det\left[X_{ik}(N_n) - \left(\frac{v^2}{(1 + i\delta)^2} - \frac{\mathbf{NSN}^{\mathrm{T}}}{\rho}\right)\delta_{ik}\right] = 0. \tag{15}$$

Eqs. (13) and (15) represent a conventional 3×3 complex-valued eigenvalue problem. We denote the complex-valued eigenvalues of the Christoffel matrix $X_{ik}(N_n)$ by $G^{(m)}(N_n)$, $m = 1, 2, 3$. These eigenvalues can be determined from the characteristic equation

$$\det[X_{ik}(N_n) - G^{(m)}(N_n)\delta_{ik}] = 0. \tag{16}$$

Using conventional approaches, we can determine eigenvalues $G^{(1)}(N_n)$, $G^{(2)}(N_n)$ and $G^{(3)}(N_n)$ from (16). These eigenvalues correspond to the three homogeneous plane waves ($qS1, qS2, qP$), which can propagate in the viscoelastic anisotropic medium under initial stresses in the direction of $\mathbf{N}$. From (15) and (16),$G^{(m)}(N_n)$ are related to $v^{(m)}$ and $\delta^{(m)}$ as follows:

$$G^{(m)}(N_n) = \frac{v^{(m)^2}(N_n)}{[1 + i\delta^{(m)}(N_n)]^2} - \frac{\mathbf{NSN}^{\mathrm{T}}}{\rho}. \tag{17}$$

Consequently, once $G^{(m)}(N_n)$ have been determined, we can simply determine the three relevant phase velocities $v^{(m)}(N_n)$, attenuation amplitude ratios $\delta^{(m)}(N_n)$, and slowness vectors $p^{(m)}(N_n)$.

From Eq. (17),

$$\frac{1}{v^{(m)}} = \frac{\mathrm{Re}\sqrt{\left[G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho\right]}}{\left|G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho\right|} \tag{18}$$

and

$$\frac{\delta^{(m)}}{v^{(m)}} = \frac{-\mathrm{Im}\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho}}{\left|G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho\right|}. \tag{19}$$

Making use of (18) in (19),

$$\delta^{(m)} = \frac{-\mathrm{Im}\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho}}{\mathrm{Re}\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho}}. \tag{20}$$

Here, the square root $\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho}$ is determined in such a way that its real part is positive.

The slowness vector $p^{(m)}(N_n)$ of a plane homogeneous wave is then given by the relation

$$p_i^{(m)}(N_n) = N_i \left[\frac{\mathrm{Re}\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho} - t\,\mathrm{Im}\sqrt{G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho}}{\left|G^{(m)}(N_n) + \mathbf{NSN}^{\mathrm{T}}/\rho\right|}\right]. \tag{21}$$

Eqs. (18), (20) and (21) represent the final results for plane homogeneous waves propagating in an arbitrary direction in an arbitrary viscoelastic anisotropic medium under the effect of initial stress.

5. Componental and mixed specifications of the slowness vector

In the componental specification, slowness vector $\mathbf{p}$ is expressed in terms of a known real-valued unit vector $\mathbf{n}$ (arbitrarily specified direction), and a complex-valued vector $\mathbf{P}^{\Sigma}$ as follows:

$$\mathbf{p} = \sigma\mathbf{n} + \mathbf{p}^{\Sigma} \tag{22}$$

with

$$\mathbf{p}^{\Sigma} \bullet \mathbf{n} = 0. \tag{23}$$

In fact, $\mathbf{p}^{\Sigma}$ represents a known vectorial component of slowness vector $\mathbf{p}$ in the plane Σ perpendicular to $\mathbf{n}$. In case of a homogeneous plane wave with the propagation vector along $\mathbf{n}$, we put

$$\mathbf{p}^{\Sigma} = 0, \tag{24}$$

where $\mathbf{0}$ denotes the null vector. In this case, Σ represents both the plane of a constant phase and the plane of constant amplitude. Consequently,

$$\mathbf{n} = \pm\mathbf{N}.$$

Waves propagating to both sides of Σ are considered, so that $\mathbf{N}$ coincides with $\mathbf{n}$, or is opposite to it. The componental specification (22) of slowness vector $\mathbf{p}$ then reduces to

$$\mathbf{p} = \sigma\mathbf{n}. \tag{25}$$

The mixed specification of the slowness vector is a special case of (22), in which $\mathbf{p}^{\Sigma}$ is purely imaginary,

$$\mathbf{p}^{\Sigma} = i\mathbf{d} \quad \text{with} \quad \mathbf{d} \bullet \mathbf{n} = 0. \tag{26}$$

For homogeneous plane wave propagating along $\mathbf{n}$, we put

$$\mathbf{d} = \mathbf{0}.$$

So, slowness vector $\mathbf{p}$ takes the same form as given by (25). On comparing Eq. (25) with Eq. (11), we obtain the result as

$$\sigma \equiv \frac{1 + i\delta}{v}. \tag{27}$$

So, in case of a plane homogeneous wave, the slowness vector $\mathbf{p}$ obtained by componental as well as mixed specifications will be the same as obtained by directional specification.

6. Special cases

6.1. Homogeneous plane waves in viscoelastic anisotropic media in the absence of initial stress

In the absence of initial stress,

$$\mathbf{X} = \frac{\mathbf{Z}}{\rho}, \tag{28}$$

$$v^{(m)} = \frac{\left|G^{(m)}\right|}{\mathrm{Re}\sqrt{G^{(m)}}}, \tag{29}$$

$$\delta^{(m)} = \frac{-\mathrm{Im}\sqrt{G^{(m)}}}{\mathrm{Re}\sqrt{G^{(m)}}}, \tag{30}$$

$$\sigma^2 = \frac{1}{G^{(m)}} \tag{31}$$

and

$$p_i^{(m)} = N_i\left[\frac{\mathrm{Re}\sqrt{G^{(m)}} - i\,\mathrm{Im}\sqrt{G^{(m)}}}{\left|G^{(m)}\right|}\right], \tag{32}$$

which are the same as obtained by Červený and Pšenčik [19].

6.2. Perfectly elastic anisotropic/isotropic medium in the presence/absence of initial stress

In case of perfectly elastic anisotropic/isotropic medium in the presence of initial stress, elastic constants B_{ijkl} will be real-valued and hence Christoffel matrix $X_{ik}(N_n)$ given by (14), is real-valued, symmetric and positive definite. Consequently, its eigenvalues $G^{(m)}(N_n), \quad m = 1, 2, 3$, are real valued and positive. Eq. (20) then implies that the attenuation amplitude factor $\delta^{(m)}$ is zero. Consequently, the attenuation vector also vanishes.

From the results deduced above, we can conclude that in a perfectly elastic anisotropic/isotropic medium in the presence/absence of initial stress, plane homogeneous waves with a non-vanishing attenuation vector parallel to the propagation vector cannot propagate.

7. Numerical results and discussion

The phase velocity, attenuation–amplitude factor are to be calculated for a given (arbitrary) phase direction. Paragneiss, a general anisotropic crystalline rock is chosen as the medium. The density of the medium is $2727\,\mathrm{kg/m^3}$. The elastic matrix (GPa) for Paragneiss (Rasolofosaon and Zinszner [20]) is given by

$$\begin{aligned}
&a_{11} = 106.8,\ a_{12} = 27.10,\ a_{13} = 9.68,\ a_{14} = -0.03Z_1,\ a_{15} = 0.28Z_1,\\
&a_{16} = 0.12Z_2;\ a_{22} = 99.00,\ a_{23} = 18.22,\ a_{24} = 1.49Z_1,\ a_{25} = 0.13Z1,\\
&a_{26} = -0.58Z_2,\ a_{33} = 54.57,\ a_{34} = 2.44Z_1,\ a_{35} = -1.69Z_1, a_{36} = -0.75Z_2,\\
&a_{44} = 25.97,\ a_{45} = 1.98Z_2,\ a_{46} = 0.43Z_1,\ a_{55} = 26.05,\ a_{56} = 1.44Z_1,\ a_{66} = 37.82.
\end{aligned}$$

where a_{ij} are the components of square matrix of order six. For the effect of viscoelastic medium, take $b_{ij} = (1+0.01i)\ a_{ij}$ to represent the complex elastic constant tensor B_{ijkl} in two suffix notation (Crampin [21]). The assumed values in matrix $\mathbf{S} = \{5, 0.6Z_2, 0.45Z_1; 0.6Z_2, 5.5, 0.5Z_1; 0.45Z_2, 0.5Z_1, 4.5\}$, defines the amount (in GPa) of initial stress in the medium. The values of $Z_1 = Z_2 = 1$ define the anisotropic medium of arbitrary (triclinic (TCS)) type. The values $Z_1 = 0$, $Z_2 = 1$ represent the monoclinic (MCS) symmetry and $Z_1 = Z_2 = 0$ represent the orthorhombic (ORS) symmetry, in the viscoelastic anisotropic medium considered.

Using the above numerical values, phase velocities of all the quasi-waves are calculated both in the presence and absence of initial stress. The difference between the two quantities represents the effect of presence of initial stress on the phase velocities. Similarly, the effects of initial stress are observed on attenuation amplitude factors of quasi-waves. The phase direction (θ, ϕ) is considered to be varying from (0,0) to (90°, 90°). Variations of these effects with phase direction are plotted in Figs. 1–3. Deviations in velocities and attenuation amplitude factors are calculated in percent. Details are as follows.

The two rows of plots in Fig. 1 represent the percent change in phase velocities and attenuation amplitude factors of qP-wave, due to the presence of initial stress. The first column of plots correspond the initial stress effect when the viscoelastic medium is viscoelastic with ORS symmetry. The second and third columns correspond to the media with MCS and TCS symmetry of viscoelastic anisotropy, respectively. It is observed from the plots that there is negligible change in phase velocities for qP-wave. For $qS1$-wave, phase velocities are changing around 1.5 percent and for $qS2$-wave these are changing upto 3%. Hence, these changes are

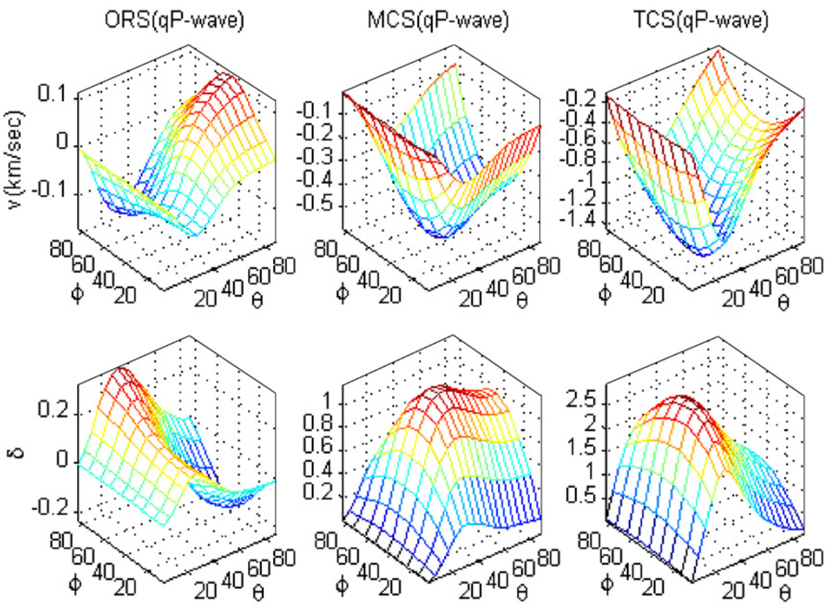

Fig. 1. Variations of percent deviations in phase velocity (v) and attenuation–amplitude factor (δ) of qP-wave for orthorhombic (ORS), monoclinic (MCS) and triclinic (TCS) viscoelastic anisotropic Paragneiss rock.

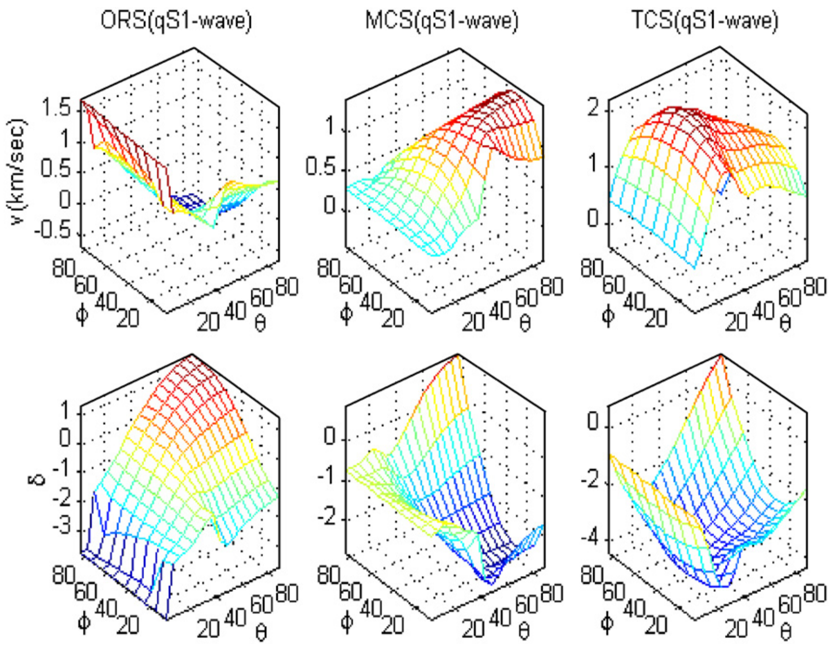

Fig. 2. Variations of percent deviations in phase velocity (v) and attenuation–amplitude factor (δ) of $qS1$-wave for orthorhombic (ORS), monoclinic (MCS) and triclinic (TCS) viscoelastic anisotropic Paragneiss rock.

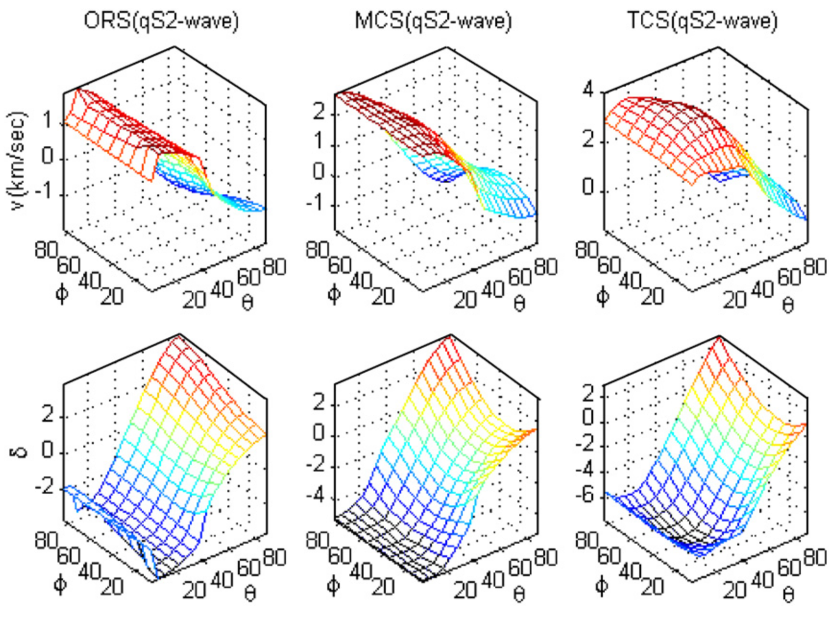

Fig. 3. Variations of percent deviations in phase velocity (v) and attenuation–amplitude factor (δ) of qS2-wave for orthorhombic (ORS), monoclinic (MCS) and triclinic (TCS) viscoelastic anisotropic Paragneiss rock.

increasing with the viscoelastic anisotropy. The directional variations of changes in the phase velocities and attenuation amplitude factors of $qS1$-wave and $qS2$-wave are presented in Figs. 2 and 3, respectively. In case of qP-wave, attenuation–amplitude factor change is upto 0.4% for ORS, 0.8% for MCS and 2% for TCS viscoelastic anisotropies. In case of $qS1$-wave, change in attenuation–amplitude factor is upto 1.5% approximately for all types of anisotropies. In case of $qS2$-wave, change in attenuation–amplitude factor is upto 4% approximately for all types of anisotropies. It is observed that the magnitude of the deviations for $qS1$-wave is slightly more than for qP-wave. Similarly, the effect of presence of initial stress on $qS2$-wave is little more than on $qS1$-wave.

The above discussion may be interpreted for the following results:

(i) The effect of initial stress on wave propagation increases with the increase of anisotropy in the viscoelastic medium.
(ii) The effect of initial stress is least on P-wave propagation and largest on $qS2$-wave propagation.
(iii) The attenuation is more sensitive to the presence of initial stress as compared to the propagation velocities.

8. Conclusions

The numerical results obtained above reveal that the effect of initial stress on plane homogeneous wave propagation in viscoelastic anisotropic medium cannot be neglected. Sharma and Neetu [11] analyzed the wave velocities in a pre-stressed anisotropic elastic medium where there was no attenuation and hence the question regarding the effect of initial stress on attenuation of plane waves is still there. For initial consideration of the attenuation of stress waves, the theory of viscoelasticity is very important. Keep it in mind, the author discussed the homogeneous wave propagation in a pre-stressed viscoelastic anisotropic medium. Červený and Pšenčik [19] presented an approach to discuss the plane waves in viscoelastic medium. They ignored the presence of initial stress in the medium. The present paper introduced the initial stress in viscoelastic anisotropic medium and concluded that the effect of initial stress on wave propagation increases with the increase of anisotropy in the viscoelastic medium. Initial stress present in the medium does not affect all the waves equally but their effect on P-wave propagation is minimum and on $qS2$-wave propagation is maximum. One more important conclusion is that the attenuation is more sensitive to the presence of initial stress as compared to the propagation velocities. Taking into account the initial stress along with viscoelastic medium will give us new ways to examine in situ stress-geometry of the sediments and reservoir rocks. The numerical discussion and results discussed above are obtained for a particular model with hypothetical initial stress values. So, these results may not quality for generalization. However, the mathematical model derived in this work may be used to compute the exact effects of initial stress on wave propagation. The viscoelasticity in the model should be viewed as means of seismic exploration in shallow consolidated sediments, reservoir rocks, zones of partial melting, etc. Initial stress in the model represents in situ stress distribution. When supported with a real/synthetic data, the mathematical model can be used for a variety of problems in geophysical, mechanical, electronics and biomedical fields. Few of them may be explained as follows:

- In the study of the postglacial isostatic readjustment process, the earth is usually modeled as viscoelastic, initially stressed, self-gravitating, radially stratified and spherically symmetric. The introduction of initial stress in a viscoelastic medium will help to establish more realistic results for the earth model.
- The study of creep in a viscoelastic body and loss of stress in the viscoelastic body and loss of stress in the viscoelastic reinforcing where a constant external load has already been applied at a certain moment, form a theoretical basis for designing, fabricating, and testing parts made of materials such as prestressed plastics (Bulavs and Skudra [22]).
- Stress fibers in living cells behave as viscoelastic cables that are tensed through the action of actomyosin motors. It is required to quantify their retraction kinetics in situ, and to explore their contribution to overall mechanical stability of the cell and interconnected extra cellular matrix (ECM) (Sanjay et al. [23]). Thus, study of viscoelastic medium under initial stress can help the biologists to a great extent.
- Viscoelastic analysis of non-prestressed or fully prestressed composite continuous beams with flexible shear connectors was studied by Dezi and Tarantino [24]. They consider a non-realistic assumption that the slab everywhere through the beam is uncracked. There is a need of realistic models to explain the viscoelastic analysis of prestressed composite beams. Hence, involving initial stress in viscoelastic medium will explore new ways for analysis of cracked or prestressed viscoelastic materials.

- In recent years, there has been significant interest in the soft materials with potential applications in bioMEMS for comfortable cancer detection and treatment. A novel shear assay technique and micro patterned biomaterial surfaces can be used to characterize cell adhesion, viscoelastic properties, and pre-stress of the human asteosarcoma cells on biocompatible surfaces, in an effort to develop tools for characterizing cancer cell properties. Therefore, some of the human organs exhibit viscoelastic properties along with initial stress and demands results for initially stressed viscoelastic medium.
- Damping augmentation is a common approach to vibration control in structures (vibroacoustic, vibration fatigue). Viscoelastic materials can be used to design efficient damping treatments. The mechanical properties of these materials however depend on frequency, but also on prestress and temperature. The study of viscoelastic medium under the effect of initial stress will help to design more efficient damping treatments.
- Stress relaxation in prestressed laminates with viscoelastic matrices, together with creep deformation under constant rate loading or large changes in temperature can be used to develop a technique leading to improvement of damage and penetration resistance of laminated composite structures, such as Army Vehicles and their armor. Applications of fiber prestress can be explored in compressive prestressing of ceramic/FRp armor plates for improved resistance to projectile penetration.
- Vibration control in machines and structures for situations involving vibration excitations can be carried out by incorporating viscoelastic materials and applying the principle of vibratory energy dissipation due to damping as a result of deformation of viscoelastic materials (Nakra [25]). Recently, Rao [26] discussed recent applications of viscoelastic damping for noise control in automobiles and commercial airplanes.

In the present paper, the author considered the case of plane homogeneous wave in a pre-stressed viscoelastic medium. A detailed study of plane inhomogeneous wave in a pre-stressed viscoelastic medium may be the subject of further studies.

Appendix A

Consider a general anisotropic pre-stressed viscoelastic medium represented by the elastic tensor B_{ijkl} and density ρ. A non-symmetric square matrix of order six, $B = \{b_{ij}\}$, is used to denote the viscoelastic properties of the medium in two –suffix notations. The asymmetry of matrix B is explained through the relations:

$$\begin{aligned}
&b_{12} - b_{21} = S_{22} - S_{11}, b_{13} - b_{31} = S_{33} - S_{11}, b_{23} - b_{32} = S_{33} - S_{22},\\
&b_{14} - b_{41} = b_{24} - b_{42} = b_{34} - b_{43} = S_{23} - S_{32}, b_{15} - b_{51} = b_{25} - b_{52} = b_{35} - b_{53} = S_{13},\\
&b_{16} - b_{61} = b_{26} - b_{62} = b_{36} - b_{63} = S_{12}, b_{54} = b_{45}, b_{64} = b_{46}, b_{65} = b_{56},
\end{aligned}$$

where S_{ij} $(i, j = 1, 2, 3)$ are the components of initial-stress tensor.

Define a row matrix $\mathbf{p} = (p_x, p_y, p_z)$, where p_j denotes the components of the slowness vector $\mathbf{p}$. A square matrix $\mathbf{Z}$ of order three is defined through its elements given by

$$\begin{aligned}
&Z_{11} = \mathbf{pAp}^{\mathrm{T}}, Z_{22} = \mathbf{pBp}^{\mathrm{T}}, Z_{33} = \mathbf{pCp}^{\mathrm{T}}, Z_{12} = \mathbf{pDp}^{\mathrm{T}}, Z_{13} = \mathbf{pEp}^{\mathrm{T}},\\
&Z_{21} = \mathbf{pFp}^{\mathrm{T}}, Z_{23} = \mathbf{pGp}^{\mathrm{T}}, Z_{31} = \mathbf{pHp}^{\mathrm{T}}, Z_{32} = \mathbf{pJp}^{\mathrm{T}},
\end{aligned} \tag{A.1}$$

where $\mathbf{p}^{\mathrm{T}}$ is the transpose of **p. A, B, C, D, E, F, G, H, J,** the square matrices of order three, are defined as follows:

$$\begin{aligned}
&\mathbf{A} = \{b_{11}, b_{16}, b_{15}; b_{61}, b_{66}, b_{65}; b_{51}, b_{56}, b_{55}\};\ \mathbf{B} = \{b_{66}, b_{62}, b_{64}; b_{26}, b_{22}, b_{24}; b_{46}, b_{42}, b_{44}\},\\
&\mathbf{C} = \{b_{55}, b_{54}, b_{53}; b_{45}, b_{44}, b_{43}; b_{35}, b_{34}, b_{33}\};\ \mathbf{D} = \{b_{16}, b_{12}, b_{14}; b_{66}, b_{62}, b_{64}; b_{56}, b_{52}, b_{54}\},\\
&\mathbf{E} = \{b_{15}, b_{14}, b_{13}; b_{65}, b_{64}, b_{63}; b_{55}, b_{54}, b_{53}\};\ \mathbf{F} = \{b_{61}, b_{66}, b_{65}; b_{21}, b_{26}, b_{25}; b_{41}, b_{46}, b_{45}\},\\
&\mathbf{G} = \{b_{65}, b_{64}, b_{63}; b_{25}, b_{24}, b_{23}; b_{45}, b_{44}, b_{43}\};\ \mathbf{H} = \{b_{51}, b_{56}, b_{55}; b_{41}, b_{46}, b_{45}; b_{31}, b_{36}, b_{35}\},\\
&\mathbf{J} = \{b_{56}, b_{52}, b_{54}; b_{46}, b_{42}, b_{44}; b_{36}, b_{32}, b_{34}\}.
\end{aligned} \tag{A.2}$$

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